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Twisted symmetry breaking on the projective hypersphere: a model of the small cosmological constant

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Abstract. A model is considered in which the gravitational action effectively depends upon the symmetry-broken vacuum state associated with a twisted scalar field theory. It is suggested that our observation of a small cosmological constant, despite predicted vacuum contributions typically of the order $10^{-6}\ \mathrm{cm}^{-2}$, is a consequence of our location in the universe.

1. Introduction

In a recent paper (Davies and Unwin 1981, hereafter referred to as I), a model was considered in which the cosmological 'constant' depended upon the symmetry-broken vacuum state of a Goldstone-type theory where the constituent scalar field carried a non-trivial representation of the group of the underlying space–time manifold (see Isham (1981) for a discussion of topology and symmetry breaking). The only constant twisted scalar field is that which vanishes globally and consequently the cosmological term was, in the symmetry-broken phase of the field theory, position dependent. It was suggested that such a mechanism could account for the small cosmological constant (the upper limit, consistent with observation, being $10^{-57}\ \mathrm{cm}^{-2}$) despite the fact that modern Higgs–Goldstone theories would predict a typically microphysical value for this quantity (see, for example, Coleman and De Luccia 1980). The twisted field theory considered in I implied that the cosmological constant, or more precisely the cosmological field, *is* typically of a microphysical value, yet of necessity we inhabit an atypical region of the universe where the value is close to zero.

The analysis in I pertained to a two-dimensional space–time in which the exact vacuum solution for the self-interacting field equation was known, and this merely gave a qualitative impression of the type of behaviour to be expected in a more realistic four-dimensional space–time model. Here, we consider just such a model in which the spatial sections of the space–time are isotropic, locally isometric to the three-sphere and yet admit non-trivial real scalar field configurations.

We begin by presenting details of the mechanism which leads to the possibility of position-dependent cosmological, and indeed Newtonian, gravitational G terms, starting with the total classical action S, where

$$S = S_1 + S_2 + S_3, \tag{1a}$$

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$$S_1 = \int d^4x \sqrt{-g}(\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{2}\xi R\phi^2 - \tfrac{1}{4}\lambda\phi^4), \tag{1b}$$

$$S_2 = \int d^4x \sqrt{-g}\mathscr{L}(A_\mu, \psi, \ldots) \tag{1c}$$

and

$$S_3 = \frac{1}{16\pi G_0}\int d^4x \sqrt{-g}(R - 2\Lambda_0). \tag{1d}$$

As usual in such models of symmetry breaking, the mass term in S_1 assumes the wrong sign, and for more generality we include a term coupling the real scalar field, ϕ, to the scalar curvature of the space–time, R, such that when $m = 0$ and $\xi = \frac{1}{6}$, the resultant wave equation for ϕ is conformally invariant. S_2 is the action for all other matter fields present, while S_3 is the gravitational action in which Λ_0 and G_0 are constants. As usual, $g \equiv \det g_{\mu\nu}$.

The vacuum solution of the field equation resulting from the variation of S_1 with respect to ϕ, we denote as σ, and define a new field variable measured from the vacuum:

$$\phi' \equiv \phi - \sigma. \tag{2}$$

Variation of S with respect to the metric then yields the gravitational field equations

$$T_{\mu\nu}(\phi', A_\mu, \psi, \ldots) = -(8\pi G)^{-1}(g_{\mu\nu}\Lambda + R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R) + X_{\mu\nu} \tag{3}$$

where

$$G = F(\sigma)G_0, \tag{4a}$$

$$\Lambda = F(\sigma)\Lambda_0 + 2\pi F(\sigma)G_0[2(4\xi - 1)\partial_\alpha\sigma\,\partial^\alpha\sigma + 8\xi\sigma\Box\sigma - 2m^2\sigma^2 + \lambda\sigma^4], \tag{4b}$$

$$X_{\mu\nu} = (2\xi - 1)\partial_\mu\sigma\,\partial_\nu\sigma + 2\xi\sigma\nabla_\mu\,\partial_\nu\sigma, \tag{4c}$$

$$F(\sigma) = (1 - 8\pi\xi G_0\sigma^2)^{-1}, \tag{4d}$$

$R_{\mu\nu}$ is the Ricci tensor and ∇_μ a covariant derivative. Our justification for selecting this particular arrangement of equation (3) is as follows.

In such models, the vacuum, σ, is interpreted as a classical 'condensate' of scalar particles (Kirzhnits and Linde 1976) and the field representing the matter present, that is, the field to be quantised, is ϕ'. Hence, equation (3) is arranged such that the potentially quantum fields (those contributing to the stress tensor) are on the left. We observe that equation (3) is of the form of the Einstein field equations with an additional term, $X_{\mu\nu}$, present as a consequence of enriched geometrical structure induced by the possible space–time inhomogeneity of the vacuum. From equations (3) and (4) it is not difficult to see that in the symmetric phase of the scalar field theory, $\sigma = 0$, Λ_0 and G_0 may be identified with the cosmological and Newtonian constants respectively, whereas in general their roles are taken by the fields Λ and G. Such models have been considered by Kirzhnits and Linde (1976), Canuto and Lee (1977), Linde (1980) and Davies (1981), where the vacuum state may be a function of time through a temperature dependence; however, these untwisted field theories shed little light upon the problem of such precise cancelling in the symmetry-broken phase (assumed to prevail today), between the symmetric phase cosmological constant, Λ_0, and the vacuum contributions (typically -10^{-6} cm^{-2} for the Weinberg–Salam model) to obtain an upper bound on Λ of 10^{-57} cm^{-2}.

2. The twisted vacuum state

The initial problem is to select a space–time, the underlying manifold of which admits twisted field configurations, yet which is not of such a perverse topological structure as to call immediately for its exclusion from the list of candidates for a plausible model of the real universe. The space–time chosen for our investigations is $\mathbb{R}^1(\text{time})\otimes P^3(\text{space})$ where P^3, the projective three-sphere, is obtained from S^3 by identifying antipodal points. It is one of a more general class of spaces covered by the three-sphere known as lens spaces, $S^3/\mathbb{Z}_n$ (Seifert and Threlfall 1934), upon which quantised scalar (Dowker and Banach 1978), spin-$\frac{1}{2}$ (Kennedy and Unwin 1980) and higher-spin fields (Unwin 1980) have been investigated elsewhere. All such spaces are homogeneous and for $n=2$ (P^3), the space enjoys the same global group of isometries, SO(4), as S^3. Despite the physically realistic properties of this space–time, which is not only locally indistinguishable from the Einstein universe, but also isotropic, we observe that

$$H^1(P^3, \mathbb{Z}_2)\approx \mathbb{Z}_2 \tag{5}$$

where $H^1(M, \mathbb{Z}_2)$ labels the inequivalent real line bundles over the manifold M, and hence conclude that $\mathbb{R}^1\otimes P^3$ admits twisted field configurations (Isham 1978).

Goldstone theories have been explored at a quantum level in the Einstein universe (Toms 1980), a slightly anisotropic or 'squashed' version of $\mathbb{R}^1\otimes S^3$ (Critchley and Dowker 1982) and quotient space–times of $\mathbb{R}^1\otimes S^3$ (Kennedy 1981), the important difference between these analyses and ours being that the symmetry-broken vacuum states of untwisted field theories (the only types allowed in the former two cases and the only types dealt with in the last) may be constants, whereas the antiperiodicity conditions associated with twisted fields exclude the possibility of non-zero constant vacua. We should mention here that Toms (1981) has investigated a twisted ϕ^4 theory on $\mathbb{R}^1\otimes P^3$ in which symmetry breaking did not occur, thus precluding the question of non-constant vacua.

Our starting point is the action S_1 of equation ($1b$), where it is now understood that ϕ is a twisted field and the integral is over the space–time $\mathbb{R}^1\otimes P^3$. A derivation of the exact symmetry-broken vacuum state for ϕ would call for a knowledge of the solutions of the field equation resulting from S_1; however there is a particularly elegant method, due to Banach (1981), for approximating the twisted vacuum, the validity of this approximation increasing as the phase transition is approached. Encouraged by the impressive correlation, at a classical level, between the results of this method and the previously known exact vacuum solutions derived by Avis and Isham (1978) for a two-dimensional space–time model, we adopt the technique in our four-dimensional analysis, referring the reader to the original paper for a detailed account of the method.

It is convenient to consider our field theory defined upon a multiply connected space–time as an automorphic field theory on the covering, $\mathbb{R}^1\otimes S^3$, (Dowker and Banach 1978) where the global constraints imposed are

$$\phi(x\gamma, t) = -\phi(x, t). \tag{6}$$

Here, x is a point on S^3, γ generates an SO(4) rotation to the antipodal point and the minus sign, a one-dimensional representation of the group factoring S^3 to obtain P^3, indicates the condition that the field be twisted. Note that, strictly, we should distinguish between the field defined upon the quotient space–time, $\mathbb{R}^1\otimes P^3$, and that defined upon the covering; however, no ambiguities should arise as a consequence of this laxity. The vacuum state for this model may now be approximated to be proportional to the static eigenfunction corresponding to the lowest eigenvalue of the space–time derivative part of the second functional derivative of the action, S_1. The approximate vacuum state, $\tilde{\sigma}$, is therefore proportional to the real static solution of

$$(\partial^2/\partial t^2 - \Delta_2)f = Kf, \tag{7}$$

where Δ_2 is the Laplace–Beltrami operator on the spatial section, such that K is minimised while (6) is respected. For this particular case, the f's are no more than the eigenfunctions of the operator associated with the field equation derived from the quadratic parts of S_1, since the inclusion of the mass and conformal coupling terms on the left of (7) (the space being of constant curvature) would only serve to shift all the eigenvalues by the same amount. Were the curvature not constant, whether conformal terms should be included in the eigenvalue equation may presumably be ascertained by example. We adopt the standard hyperspherical coordinate system of line element

$$ds^2 = dt^2 - a^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\beta^2)] \tag{8}$$

where a is the radius of S^3, and find for the relevant four K-degenerate solutions of (7),

$$\tilde{\sigma}_1 = A_1 \cos\chi, \tag{9a}$$

$$\tilde{\sigma}_2 = A_2 \sin\chi \cos\theta, \tag{9b}$$

$$\tilde{\sigma}_3 = A_3 \sin\chi \sin\theta \cos\beta, \tag{9c}$$

$$\tilde{\sigma}_4 = A_4 \sin\chi \sin\theta \sin\beta. \tag{9d}$$

It would appear, at first glance, that we have four different possible candidates for the vacuum solution; however, we recognise that if A_i were replaced by a, $\tilde{\sigma}_i$ would be no more than the familiar cartesian coordinates associated with the embedding of S^3 in four-dimensional Euclidean space. That is, the $\tilde{\sigma}_i$ depend purely upon the radial (geodesic) distance from a point (different for each i) on P^3. The four vacua are therefore entirely equivalent via spatial rotations, and indeed, being at liberty to orientate the hyperspherical coordinate systems in any manner we wish, we deduce that there is an infinite number of equivalent candidates for the vacuum state, each depending only upon the radial distance from a different point on P^3, henceforth referred to as the 'centre'. This is precisely analogous to the situation considered by Avis and Isham (1978) in which there exists an arbitrary phase along the spatial circle in their twisted vacuum field solution on the space–time $\mathbb{R}^1 \otimes S^1$.

Having established the spatial dependence of the possible vacua, we proceed to determine the magnitude of $\tilde{\sigma}$, and in accordance with the work of Banach, generalise the usual definition of the effective potential to encompass the twisted field sector. The generalised definition is

$$V(A) = -(\mathrm{VOL})^{-1}\Gamma(\tilde{\sigma}), \tag{10}$$

where A is the amplitude of $\tilde{\sigma}$, VOL is the space–time volume and $\Gamma(\tilde{\sigma})$ is the effective action, evaluated at $\tilde{\sigma}$. Without loss of generality, we choose the 'centre' to coincide with $\chi = 0$ such that $\tilde{\sigma} = A\cos\chi$, and at the tree graph level (to which we confine our attention here), $\Gamma(\tilde{\sigma})$ is obtained by evaluating S_1 at $\phi = \tilde{\sigma}$ to yield the expression

$$V(A) = \tfrac{1}{32}[\lambda A^4 - 4m^2A^2 + 12A^2(2\xi + 1)/a^2]. \tag{11}$$

We discover that if $a \leqslant m^{-1}[3(2\xi + 1)]^{1/2}$, the minimum of V occurs at $A = 0$, otherwise occurring for some finite value of A, or, more precisely, the approximate vacuum solution is given by

$$\tilde{\sigma} = 0 \qquad \text{if } a \leqslant m^{-1}[3(2\xi+1)]^{1/2}, \tag{12a}$$

$$\tilde{\sigma} = A' \cos\chi \qquad \text{if } a \geqslant m^{-1}[3(2\xi+1)]^{1/2}, \tag{12b}$$

where

$$A' = (1/a)(2/\lambda)^{1/2}[m^2a^2 - 3(2\xi+1)]^{1/2}. \tag{12c}$$

The field theory, we see, displays a phase transition at some critical radius, the vacuum, $\tilde{\sigma}$, being uniformly continuous in a. To determine exactly how good an approximation to the true vacuum solution $\tilde{\sigma}$ is, we need only reinsert the latter into the wave equation for ϕ in order to evaluate the associated source current, J. We have, in general, with a source present,

$$(\Box - m^2 + \xi R + \lambda\phi^2)\phi = J(\phi) \tag{13}$$

where, of course, for the true vacuum solution, $J = 0$. The insertion of $\tilde{\sigma}$ into (13) yields the expression

$$J(\tilde{\sigma}) = \tfrac{1}{2}\lambda A^3 \cos\chi \cos 2\chi \tag{14}$$

and we conclude that $\tilde{\sigma} = 0$ ($A = 0$) is the exact symmetric phase vacuum, whereas $\tilde{\sigma}$ is an increasingly good approximation to the true symmetry-broken vacuum as $A = A' \to 0$, that is, as the phase transition is approached. Banach (1981) argues that in cases where the vacuum solution is uniformly continuous through the phase transition, the critical radius should be exact since at this radius, the vacuum is exactly zero and small deviations of $\tilde{\sigma}$ from the true vacuum at slightly greater radii are insignificant to the order considered.

It may be of interest to compare equation (12) with the vacuum stability criteria associated with the untwisted field theory of action S_1 on $\mathbb{R}^1 \otimes P^3$. The vacuum is classically exact and identical to the $\mathbb{R}^1 \otimes S^3$ case:

$$\sigma = 0 \qquad \text{if } a \leqslant m^{-1}(6\xi)^{1/2}, \tag{15a}$$

$$\sigma = (1/a)(1/\lambda)^{1/2}(m^2a^2 - 6\xi)^{1/2} \qquad \text{if } a \geqslant m^{-1}(6\xi)^{1/2}. \tag{15b}$$

We observe that at this classical level the untwisted field theory, unlike the corresponding twisted theory, requires a non-zero conformal coupling constant, ξ, for there to exist a symmetric, $\sigma = 0$, phase.

3. The gravitational field equations

It is now straightforward to obtain approximate expressions for the effective Newtonian and cosmological fields along with $X_{\mu\nu}$ in the symmetry-broken phase of the twisted theory, it being understood that the approximation improves as the transition is approached. The insertion of (12b) into (4) yields

$$G = F(A' \cos\chi)G_0, \tag{16a}$$

$$\Lambda = F(A'\cos\chi)\Lambda_0 + 4\pi a^{-2} G_0 A'^2 F(A'\cos\chi) \times[\cos^4\chi(m^2a^2 - 3 - 6\xi) + \cos^2\chi(16\xi - 1 - m^2a^2) + 1 - 4\xi] \tag{16b}$$

where

$$F(A'\cos\chi)=\{1-(16\pi\xi G_0/\lambda a^2)[m^2a^2-3(2\xi+1)]\cos^2\chi\}^{-1} \tag{16c}$$

and the non-vanishing components of $X_{\mu\nu}$ are

$$X_{\chi\chi}=A'^2[(4\xi-1)\sin^2\chi-2\xi], \tag{16d}$$

$$X_{\theta\theta}=-\tfrac{1}{2}\xi A'^2\sin^2 2\chi, \tag{16e}$$

$$X_{\beta\beta}=-\tfrac{1}{2}\xi A'^2\sin^2 2\chi\sin^2\theta. \tag{16f}$$

The complete solution of equation (3) would now, in principle, entail an iterative procedure in which equation (3) is solved for the metric, the new vacuum associated with the scalar field for this metric determined, reinserted into equation (4) and the procedure repeated until self-consistency is attained.

We now investigate the forms of G, Λ and $X_{\mu\nu}$, confining our attention to the minimally coupled case ($\xi=0$), where equation (16) reduces to

$$G=G_0, \tag{17a}$$

$$\Lambda=\Lambda_0-\pi G_0a^{-2}A'^2[(m^2a^2-3)\sin^2 2\chi+4(1+2\cos 2\chi)] \tag{17b}$$

and the only non-vanishing component of $X_{\mu\nu}$ is

$$X_{\chi\chi}=-A'^2\sin^2\chi, \tag{17c}$$

A' being evaluated at $\xi=0$. There is no unique way to cover P^3 with the hyperspherical coordinate system, and one option is to allow θ and β to assume their full S^3 ranges while χ varies only from 0 to $\pi/2$. The spatial dependence of G is trivial, that of $X_{\chi\chi}$ is not difficult to visualise, and hence we direct our attention to a description of the cosmological field, Λ. We have

$$\Lambda_1(0)=-3\Lambda_1(\pi/2)=(-24\pi G_0/\lambda a^4)(m^2a^2-3) \tag{18a}$$

where

$$\Lambda_1(\chi)\equiv\Lambda(\chi)-\Lambda_0. \tag{18b}$$

If $3\leq m^2a^2<7$, the lower limit reflecting the lower bound on the range of validity of (17), then Λ_1 is maximised at $\chi=\pi/2$ and minimised at $\chi=0$. If $m^2a^2>7$, the validity of (17) in this range being discussed in the following section, then Λ_1 is maximised at both $\chi=0$ and $\pi/2$, a minimum occurring at $\cos 2\chi=4(m^2a^2-3)^{-1}$, where

$$\Lambda_1^{\min}=(-2\pi G_0/\lambda a^4)(m^4a^4-2m^2a^2+13). \tag{19}$$

For comparison, we present the classically exact expression for the cosmological constant, Λ_u, of the corresponding untwisted, $\xi=0$ field theory in the symmetry-broken phase:

$$\Lambda_u=\Lambda_0-2\pi G_0m^4\lambda^{-1}. \tag{20}$$

We mention that although, classically, there is no symmetric phase in the $\xi=0$ untwisted field theory, thermal and quantum corrections to the effective potential modify the vacuum stability criteria (see, for example, Canuto and Lee 1977).

4. Discussion

We summarise the important features of the model considered. Having introduced a twisted Goldstone field theory upon the space–time $\mathbb{R}^1 \otimes P^3$, we discovered that in the symmetry-broken phase of the former, the vacuum state did not enjoy the homogeneity and isotropy enjoyed by the underlying space–time manifold. This resulted not only in additional terms appearing in the usual gravitational field equations, but effective cosmological and Newtonian fields which depended upon the geodesic distance from some fixed point on P^3.

Our wish is to reconcile the properties of this model with present observations which favour an extremely small cosmological constant. Initially, we recognise that the radius of P^3 must be many orders of magnitude greater than the Hubble radius to ensure that Λ is approximately constant over observable distances, a fact consolidated by the absence of observed systematic gravitational disruption. This would appear to place a well outside the range over which our approximation of the vacuum state is a good one. However, the good qualitative correlation between the approximate and exact vacua in the $\mathbb{R}^1 \otimes S^1$ model considered by Banach (1981), even well away from the phase transition, encourages us to investigate further the form of equation (17*b*) for large ma, to obtain order of magnitude estimates on the range of Λ (this being all of which we are capable anyway), and to approximate its spatial dependence. Attention will be confined to the $\xi = 0$ case and from equations (18) and (19) we see that for $ma \gg 1$, Λ_1 has two sharp maxima, only becoming positive close to $\pi/2$ (crossing zero at $\cos 2\chi \approx (5 - m^2a^2)(m^2a^2 - 3)^{-1}$). Inserting the values of m and λ associated with Weinberg–Salam theory (Weinberg 1976) into equation (18*a*), we conclude that $\Lambda_1(\pi/2)$, the maximum value of Λ_1, must be positive and many orders of magnitude less than $10^{-90}\ \mathrm{cm}^{-2}$. The minimum of Λ_1, occurring at approximately $\chi = \pi/4$, has a value independent of a and equal to $\Lambda_u - \Lambda_0$, the vacuum contribution to the cosmological constant in the untwisted field theory. Again, inserting typical numbers, we find that $\Lambda_1^{\min} \sim -10^{-6}\ \mathrm{cm}^{-2}$.

In the case of the untwisted theory, it is not difficult to see where problems arise in accounting for the small cosmological constant favoured by observation. Extremely fine tuning between Λ_0, the cosmological constant in the symmetric phase, and the vacuum contributions to Λ_u would be demanded in order to obtain a realistic value of the symmetry-broken phase cosmological constant of less than $10^{-57}\ \mathrm{cm}^{-2}$. An anthropic explanation of such tuning may, of course, be offered. If such precise cancelling did not occur, we should not be here to ponder such matters since a huge cosmological constant would dominate gravitational dynamics, precluding the formation of galaxies and hence life. The twisted field theory considered here, however, suggests a rather different approach to understanding the observed small cosmological constant. The latter may more accurately be described as a cosmological field, the magnitude of which depends upon the radial distance (it being assumed that this is a property of the exact, as well as the approximate vacuum) from some point on the spherical spatial section. The only constraint we need impose upon Λ_0 is that it be in a range such that Λ is somewhere close to zero. For the approximation developed here, $\Lambda_0 = 0$ is within that range, and indeed, if we require the gravitational field equations to assume their familiar form locally, that is $X_{\mu\nu} = 0$, then $\chi \approx 0$ would be the suitable candidate for our own location where the effective cosmological constant (for $\Lambda_0 = 0$) would be many orders of magnitude less than $10^{-90}\ \mathrm{cm}^{-2}$. In this case, it is perhaps amusing to note that our model places the philosophy of anthropocentrism, an idea so vigorously rejected by the conventional scientific wisdom, in a new light.

An accurate assessment of the space dependence of Λ would, of course, require a knowledge of the exact symmetry-broken twisted vacuum state, entailing the solution of equation (13) for zero source current. However, from the approximation employed here, it is clear that in such models, altering the global structure assigned to the scalar field, even when its local dynamical constraints remain unchanged, has the effect of modifying, radically, the resultant gravitational field equations. Here, we have highlighted the aspects of a twisted field theory which are, in many ways, preferable to those of the corresponding untwisted theory.

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